Common Errors – Algebra

Each of the "common errors" shown below first demonstrates an incorrect use of algebra. Comments are made regarding why this is incorrect.

1. $\sqrt{9} = \pm 3$ **ERROR** **Use of the radical**

1a. The radical is by definition the positive square root. Thus $\sqrt{9} = +3$ and NOT -3

1b. If there is a variable under the radical, absolute value must be used to guarantee a positive result. Therefore, $\sqrt{x^2} = |x|$ and $\sqrt{(x-2)^2} = |x-2|$

2. $x^2 = 9 \rightarrow x = 3$ **ERROR** **Two roots**

2a. If $x^2 = 9$, then $x = \pm 3$. It is true that $x = 3$ is one solution. Always use <u>both</u> +3 and –3 as solutions.

2b. The complete solution is $\sqrt{x^2} = \sqrt{9} \rightarrow |x| = 3 \rightarrow \begin{cases} x = 3 \\ x = -3 \end{cases}$ because $\sqrt{x^2} = |x|$. (See 1b above)

OR change $x^2 = 9$ to $x^2 - 9 = 0 = (x+3)(x-3)$ and see the two answers ± 3 more clearly.

3. $(a+b)^2 = a^2 + b^2$ **ERROR** **Squaring two terms**

3a. This should be $(a+b)^2 = a^2 + 2ab + b^2$. Similarly, $(a-b)^2 = a^2 - 2ab + b^2$.

4. $\sqrt{a^2+b^2} = \sqrt{a^2} + \sqrt{b^2} = a + b$ **ERROR** **The square root of two terms**

4a. $\sqrt{a^2+b^2}$ cannot be simplified. Proof this is wrong: Let $a = 1$ and $b = 2$ and check $\sqrt{1^2+2^2} \neq 1+2$

$$\sqrt{5} \neq 3$$

5. $\dfrac{1}{a+b} = \dfrac{1}{a} + \dfrac{1}{b}$ **ERROR** **There is ONE denominator**

5a. $\dfrac{1}{a+b}$ cannot be simplified. Proof that this is wrong: Let $a = 1$ and $b = 2$ and check $\dfrac{1}{1+2} \neq \dfrac{1}{1} + \dfrac{1}{2}$

$$\frac{1}{3} \neq 1 + \frac{1}{2}$$

6. **ERROR** **Be careful when "cancelling"**

$$\frac{3x+y}{x^2} = \frac{3+y}{x} \quad \text{or} \quad \frac{10x-13}{25} = \frac{2x-13}{5} \quad \text{or} \quad \frac{(x-2)\bullet 4x^2-3x}{(x-2)^2} = \frac{4x^2-3x}{(x-2)}$$

6a. None of these expressions can be simplified as written. The common errors are:

$$\frac{3\cancel{x}+y}{x^{\cancel{2}}} = \frac{3+y}{x} \quad \text{or} \quad \frac{\cancel{10}x-13}{\cancel{25}} = \frac{2x-13}{5} \quad \text{or} \quad \frac{\cancel{(x-2)}\bullet 4x^2-3x}{(x-2)^{\cancel{2}}} = \frac{4x^2-3x}{(x-2)}$$

6b. $\dfrac{4x-3xy}{2x^7} = \dfrac{x(4-3y)}{2x^7} = \dfrac{4-3y}{2x^6}$ and <u>canceling works</u> because the numerator can be factored. The 4 and the 2 can NOT be cancelled because a 2 can NOT be factored in the numerator.

7. **The ZERO product property**

Solve for x:

$x(x-2)=1$

$x=1$ or $x-2=1$

$x=3$

ERROR
NOTE: This is incorrect because the right hand side of the equation is not zero, and the factors x and $(x-2)$ have been set equal to that non-zero number, i.e., 1.

7a. The zero product property states that if $ab = 0$ then either $a = 0$ or $b = 0$. This property is applicable only if the right hand side of the equation is zero.

7b. The correct solution is found by multiplying $x(x-2)=x^2-2x=1$

Then $x^2-2x-1=0$

Try to factor, but since this will not factor nicely, use the quadratic formula to solve for x.

8. Pulling a negative number out of an absolute value: $|-2(x+3)|=-2|x+3|$ **ERROR**

8a. This would mean that an absolute value expression on the left, which must be positive or 0, is equal to an expression which is negative (except when $x = -3$).

8b. It has to be done like this: $|-2(x+3)|=2|x+3|$.

9. (i) $3(2x-7)^4=(6x-21)^4$ or (ii) $5\sqrt{2x-3}=\sqrt{10x-15}$ **ERROR** **Distributive Property**

9a. Proof that this is wrong: Let $x = 4$. (i) $3(2x-7)^4=3(1)^4=3\neq(24-21)^4=81$.

Let $x = 2$. (ii) $5\sqrt{2x-3}=5\sqrt{1}=5\neq\sqrt{20-15}=\sqrt{5}$.

9b. Do NOT distribute a in $a(bx+c)^{\text{exp}}$ if the expression in parentheses involves an exponent other than 1.

10. $\left(x^2\right)^3=x^5$ **ERROR** **Using more than one exponent**

10a. $\left(x^2\right)^3$ means to use the x^2 three times as a factor: $x^2\bullet x^2\bullet x^2=x^6$. There are a total of six x's because x^2 has been used three <u>times</u>. To simplify $\left(x^2\right)^3$, multiply the exponents.

10b. It is correct that $x^2\bullet x^3=x^5$ and the exponents can be added when x^2 and x^3 are multiplied.

11. $\log_b(x+y) = \log_b(x) + \log_b(y)$ **ERROR** **Adding logarithms**

11a. Proof that this is wrong: let b = 10, x = 10 and y = 100; then $\log_{10}(x)$ = 1 and $\log_{10}(y)$ = 2
$\log_{10}(x) + \log_{10}(y) = 3$ but $\log_{10}(x+y) = \log_{10}(110) \approx 2.04$

11b. $\log_{10}(10)$ = 1 means that the <u>exponent</u> of 10 is 1; $\log_{10}(100)$ = 2 means that the <u>exponent</u> of 10 is 2. It is true that $\log_b(x \bullet y) = \log_b(x) + \log_b(y)$ because <u>exponents</u> are added when the quantities are multiplied (see 10b above).

12. $\dfrac{\log_b(x)}{\log_b(y)} = \log_b\left(\dfrac{x}{y}\right)$ **ERROR** **Division and logarithms**

12a. Proof that this is wrong: let b = 10, x = 100, and y = 1000; then $\log_b(x)$ = 2 and $\log_b(y)$ = 3
$\dfrac{\log_b(x)}{\log_b(y)} = \dfrac{\log_{10}(100)}{\log_{10}(1000)} = \dfrac{2}{3}$ but $\log_{10}\left(\dfrac{x}{y}\right) = \log_{10}\left(\dfrac{100}{1000}\right) = \log_{10}\left(10^{-1}\right) = -1$

12b. It is true that $\log_b\left(\dfrac{x}{y}\right) = \log_b(x) - \log_b(y)$ because <u>exponents</u> are subtracted when the quantities are divided. $\left(\text{Example: } \dfrac{x^7}{x^5} = x^{7-5} = x^2\right)$

13. $\dfrac{\log(3a)}{\log(2a)} = \dfrac{\log(3\not{a})}{\log(2\not{a})} = \log\left(\dfrac{3}{2}\right)$ **ERROR** **Canceling and logarithms**

13a. Proof that this is wrong: Let x = 1. Then $\dfrac{\log(3x)}{\log(2x)} = \dfrac{\log(3)}{\log(2)} \approx \dfrac{0.4771}{0.3010} \approx 1.585$ but $\log\left(\dfrac{3}{2}\right) \approx 0.1761$

There is no way to simplify the expression $\dfrac{\log(3x)}{\log(2x)}$ by canceling.